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Abstract

The paper utilizes a model of endogenous growth with vertical innovation (á la Aghion-Howitt) to examine how the inclusion of a production related pollution externality affects the prospect for long-run growth of a closed economy. It is derived that the social optimum exhibits the possibility of long-run sustainable growth, such that consumption, capital stock and output grow without bound, knowledge also grows in an unbounded fashion, and both – the intensity and stock of pollution – fall. In comparison, at the unregulated market equilibrium, a clear conflict arises between sustaining economic growth and environment protection, as growing pollution stock ceases the opportunity for long run growth in output, capital stock and consumption. Finally, in deriving the optimal public policy tool-kit, given the distortions in the unregulated market economy, it is shown that a positive and growing rate of tax on pollution, an ad valorem subsidy on capital and a positive R and D subsidy would implement the socially desirable outcome. However, a theoretical possibility of an optimal tax on R and D cannot be ruled out in an exceptional situation of too low a productivity of the R and D sector.

Journal of Economic Literature Classifications: O10, O11, O13, Q01, Q28, Q55, Q56, Q58.

Key Words: endogenous growth, innovation, vertically-differentiated inputs, pollution externality, growth-environment trade-off, optimal public policy.

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1 Introduction

Akin to economic growth, environmental protection is now recognized as an important development imperative. It is being increasingly acknowledged that regimes that emphasize economic growth while ignoring environmental damage are not sustainable in the long run. Whether environmental pollution arises in production or consumption activities, public policies need to ensure internalization of these environmental effects. Having made the transition from neoclassical paradigms, economic growth theory, which now relies on the role of endogenous technical change/ innovation, provides a powerful tool to incorporate and suggest ways to cope with the environmental effects associated with the economic growth process.

Several variants of endogenous growth frameworks have been extended to analyze how environmental constraints could be included in various ways and test the prospect for sustainable growth from a long-run perspective (see, for example, Gradus and Smulders (1993), Verdier (1993), Hung, Chang and Blackburn (1994), Bovenburg and Smulders (1995, 1996), Elbasha and Roe (1996), Aghion and Howitt (1998), Grimaud (1999), Grimaud and Ricci (1999) and Pittel (2002)). Gradus and Smulders (1993) study the effect of pollution arising from the use of capital in production and pollution causes disutility. Three sets of growth models distinguishing three alternative production structures are utilized – one is the neoclassical production function with exogenous technical change, the second is the AK production structure with constant returns to capital (due to Romer (1986) and Rebelo (1991)), and the third is the Lucas (1988) production structure where human capital accumulation drives economic growth. None of these modeling frameworks utilize the vertical innovations paradigm as in the current paper. Bovenburg and Smulders (1995, 1996) extend the endogenous growth models of Lucas (1988) and Rebelo (1991) to utilize a two-sector framework, where one sector produces a final good and the other sector generates knowledge that helps in pollution reduction, and knowledge is a public good. Since environment is a factor of production, better environmental quality spurs growth. But, technology improvement does not work through incentives to innovate. Hung, Chang and Blackburn (1993), Verdier (1993), Elbasha and Roe (1996) and Pittel (2002) analyze environmental pollution having adverse effects on utility in a model of horizontally differentiated intermediate inputs (á la Romer (1986)). These papers derive the decentralized market equilibrium and compare it with the socially desirable outcome. However, none of these explicitly characterize the policy instruments toward achieving the social optimum.

Through this theoretical inquiry we attempt to seek answers to the following set of questions. What are the implications of introducing environmental pollution in a Schumpeterian model of endogenous growth where technological change allows for improvement in the quality or productivity of the intermediate inputs? That is, does the inclusion of environment enhance or reduce the rate of long-run economic growth? Is the private competitive equilibrium Pareto optimal? If not, what policies can lead to private equilibrium to converge to the social optimum?

The paper utilizes the Aghion and Howitt (1998) framework of vertically differentiated intermediate inputs to, first, characterize the social planner's equilibrium. Next, this is compared with an unregulated, market outcome. And finally, given the three identifiable market distortions in the economy, environmental pollution, market power (in the intermediate goods' sectors) and knowledge spillovers, we characterize the first-best policy instruments for each that will implement the social optimum.

Grimaud (1999) extends the Aghion and Howitt (1998) analysis to derive the decentralized outcome, but this is done using a Romer (1986) model of horizontal innovations or expanding varieties of intermediate inputs, while also allowing for pollution permits. Grimaud and Ricci (1999) compare the growth-environment trade-off between the alternative regimes of variety expansion and quality improvements leading to endogenous growth. They characterize the optimal levels of policy instruments to correct for the discernible market distortions. So, in this respect, our work is the closest in structure to theirs. However, in our paper we first identify the need for optimum policies by highlighting the conditions under which growth at the unregulated decentralized equilibrium will be unsustainable or sustainable. Further, we more explicitly characterize the optimal first-best public policy instruments that will implement the socially desirable outcome. The analysis is extended to understand how environmental policy impacts economic growth, and how do the other two policy instruments, that is, subsidy on capital and R and D interact with environmental policy to influence prospects for sustainable growth.

The key results derived are as follows.

1. At the planner's equilibrium, the possibility of long-run sustainable growth exists, characterized by unbounded growth of consumption, capital stock, output and knowledge, and a steady fall in the intensity and stock of pollution.

2. In comparison, at the unregulated market equilibrium, a definitive conflict arises between sustaining economic growth and environment protection, as growing pollution stock rules out the possibility of long run growth in output, capital stock and consumption.

3. Given the distortions in the unregulated market economy, a positive and growing rate of tax on pollution, an ad valorem subsidy on capital and a positive R and D subsidy would implement

the socially desirable outcome. However, a theoretical possibility of an optimal tax on R and D cannot be ruled out in an exceptional situation of too low a productivity of the R and D sector.

4. Finally, a more stringent environmental policy impacts economic growth through depressing the output of the final good, reducing the marginal benefits from innovation by lowering the demand for intermediate goods, and, at the same time, lowering the marginal cost of innovation by depressing the demand for labour in the final good sector. A positive subsidy to capital counters the depressing effect on the demand for intermediates, by raising the profits of the monopoly producers, thus enhancing the marginal value of innovation, while a positive subsidy to the R and D sector further lowers the marginal cost of R and D. Both of these effects spur innovation activity to offset the dampening effect of a stricter environmental policy on economic growth.

The structure of the paper is as follows. In Section 2 the model is outlined. Section 3 characterizes the social planner's equilibrium. In Section 4 the outcomes for the unregulated market equilibrium are characterized, and compared with the social optimum. Section 5 derives and discusses the first-best public policies that will achieve the social optimum. Section 6 concludes.

2 The model framework

We consider a model of endogenous growth with vertical innovations (á la Aghion and Howitt (1992)), set up in continuous time. The economy comprises three production sectors: producers of a final good, intermediate goods producers, and R and D firms. The final good producers demand a range of intermediate goods from the intermediate goods sectors, combine it with labor and an environmental input to produce the final good. An intermediate good has a quality ladder along which improvements in its productivity can take place. Each intermediate good is supplied by a local monopoly firm, who owns the exclusive right over the design/ blueprint that is required to produce it. The monopoly combines capital rented from the households to produce these intermediate goods. The monopoly rents are not perpetual, but derived only until the next innovation arrives and makes the current design/ blueprint obsolete. The R and D firms hire labor competitively to produce an improved or a higher quality intermediate good. The production of the final good is pollution intensive, and pollution accumulation deteriorates environmental quality. However, the environment has an inherent capacity to assimilate pollution. There exists a fixed mass of population/ consumers, which does not grow over time and also constitutes the source of labor supply to the economy. All individuals are homogeneous, and the utility of the representative consumer is defined over the consumption of the final good and environmental quality. We describe the individual sectors below.

2.1 The final goods sector

The final homogeneous good (Y) is produced and sold in a competitive market. For any firm, the production structure at time t is defined as

$$Y(t) = z(t)((1-u(t))L)^{1-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^{\alpha}di, \;\; i\in[0,1],\; 0<\alpha,\; u(t)<1. \tag{1}$$

Thus, production of the final good uses raw labor, L, an environmental input, $z(t)$, and a quality indexed intermediate input, $x(i,t)$, where index i denotes the type of the intermediate input. The aggregate mass of labor does not grow in time and $(1-u(t))$ is the fraction of labor employed in its production. (The remaining fraction $u(t)$ is used in the R and D sector.) $z(t)$ is the aggregate pollution intensity of $Y(t)$, and $A(i,t)$ is the productivity index for the intermediate good i that improves over time through successive application of research effort in the ith R and D sector. The average productivity index is defined as $A(t) = \int_0^1 A(i,t)di$.

Good Y is also assumed to be the numeraire good, such that its price, $p_Y = 1$. In any period t, the output $Y(t)$ can be allocated to consumption or investment, in a one-on-one fashion. That is, capital accumulates as

$$\dot{K}(t) = Y(t) - C(t), \tag{2}$$

where $C(t)$ and $K(t)$ denote economy-wide consumption and capital stock.

The demand for the individual inputs is determined by optimizing the net profit function denoted by $\Pi_Y(t)$ in each time period.

2.2 The intermediate goods monopoly firms

In each intermediate good sector, i, there is a unique entrepreneur who buys the patent or license to produce quantity $x(i,t)$ at time t. The entrepreneur is the monopoly producer in the ith sector. The patent/ blueprint in sector i can be valid for an infinite period, but it gets obsolete upon the arrival of the next blueprint/ innovation, which replaces it (the standard Aghion-Howitt process of “creative destruction”). Further, at any point in time, knowledge exists to produce an array of qualities of each type of intermediate good, but we focus on the equilibrium in which only the leading-edge quality is actually produced by the individual intermediate good sector, and used by the final good producers to generate Y.

Each intermediate good uses only capital in a one-to-one production technology, or $x(i,t) = K(i,t)$. Thus, the amount of intermediate good produced of all types equals the aggregate capital stock of the economy, that is,

$$\int_0^1 x(i,t)di = K(t). \tag{3}$$

Until the arrival of the next innovation in design, the monopolist in the ith sector reaps a rent on the sale of commodity level $x(i,t)$. The instantaneous level of net rent of the monopolist is denoted by $\Pi_x(i,t)$.

2.3 The R and D sector

The new designs or qualities of (each of) the ith input are produced in the R and D sector. Individual firms in each research sub-sector employ a fraction of the entire labor pool, $u(i,t)L$, competitively and discover the design for the next generation of the ith intermediate good. Further, there is no restriction on the entry of new firms in R and D. Labor is the only rival input used in the R and D sector, where firms employ labor in a stochastic constant returns production function, governed by a Poisson process, with the arrival rate of new designs denoted by the parameter η. This is assumed to be equal across all the R and D sectors. Then, once the innovation is achieved, only the highest quality productivity parameter, $\bar{A}(t)$, is used. This is called as the leading edge technology in sector i, such that

$$\bar{A}(t) = max\{A(i,t)\}. \tag{4}$$

This aspect captures the intersectoral spillovers in R and D.

The growth of $A(t)$ is proportional to the Poisson arrival rate in sector i, η, the amount of labor allocated to R and D, $u(i,t)L$, and the size of the innovation, or the rate at which innovation expands the economy's technology frontier, $(\sigma - 1)$. For simplicity, we let $u(i,t) = u(t)\ \forall i$, where $u(t)$ is the fraction of labor used in the R and D activity. Thus, the instantaneous flow of innovations in the economy is given by $\dot{A}(t) = (\sigma - 1)\eta u(t)LA(t)$, or the existing stock of knowledge $A(t)$ is available in a non-rival manner to all the R and D firms. In addition, since each innovation has an equal likelihood of occurring in any sector, the leading edge parameter $\bar{A}(t)$, can be shown to grow proportional to the flow of innovations, that is,

$$\frac{\dot{\bar{A}}(t)}{\bar{A}(t)} = (\sigma - 1)\eta u(t)L, \tag{5}$$

which is derived from the process described in Appendix A.

2.4 The environment sector

The pollution arises from the production activity in the final good sector. The flow of pollution, $P(t)$, is a function of the level of output of $Y(t)$ and the index of pollution intensity, $z(t)$. Specifically, we have

$$P(t) = Y(t)z(t)^{\beta}, \quad \beta > 0, \tag{6}$$

which can be used to express the production of final good in terms of P as

$$Y(t) = P(t)^{\frac{1}{(1+\beta)}} \left[((1-u(t)L)^{1-\alpha} \int_0^1 A(i,t)^{1-\alpha} x(i,t)^{\alpha} di \right]^{\frac{\beta}{(1+\beta)}}, \tag{7}$$

implying constant returns to scale with respect to P one the one hand and rest of the inputs on the other.

The flow of pollution emanating in the final good sector accumulates and determines the environmental pollution stock, $E(t)$, which can be used as an indicator of environmental quality. However, similar to the other renewable resources, environment has an inherent ability to absorb or assimilate pollution. E rises if the pollution loading level from the production sector exceeds the amount that is absorbed/ assimilated by nature. For analytical tractability we assume the assimilation function to be linear in the level of pollution. Thus, the evolution of the stock of pollution is modeled as

$$\begin{aligned} \dot{E}(t) &= P(t) - \theta E(t), \quad 0 < \theta < 1 \\ &= Y(t)z(t)^{\beta} - \theta E(t). \end{aligned} \tag{8}$$

Similar to Aghion and Howitt (1998) and Grimaud (1999), we assume that there exists an upper bound on the pollution stock, or $\bar{E}$, such that in equilibrium E does not grow beyond this level. That is, $E(t) \in [0, \bar{E}]$, otherwise, any excessive level of pollution stock inhibits or rules out production.

2.5 The consumers

The representative consumer maximizes lifetime utility by discounting its future levels at rate ρ per period. All individuals are infinitely-lived, and have utility defined over the consumption of the final good and pollution stock in an additively separable fashion. That is,

$$U\left(c(0), \frac{E(0)}{L}\right) = \int_0^{\infty} e^{-\rho t} \left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] dt, \quad \rho > 0, \rho \neq 1, \quad \epsilon > 0, \quad \delta > 0. \tag{9}$$

In the above expression $c(t)$ and $E(t)/L$ are consumption and environmental pollution stock in per capita terms. The intertemporal elasticity of substitution with respect to the final good consumption is ϵ, and the elasticity of utility with respect to pollution is $1+\delta$. The magnitude of the latter determines the consumer's preference for clean environmental quality.

3 The planner's equilibrium

3.1 Planner's optimization

The planner's optimization program is given as

$$
\begin{aligned}
Max\, U\left(c(0), \frac{E(0)}{L}\right) &= \int_0^\infty e^{-\rho t}\left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] dt \\
\text{s.t.} & \\
\dot{K}(t) &= z(t)((1-u(t))L)^{1-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^\alpha di - c(t)L \\
&= z(t)((1-u(t))L)^{1-\alpha}A(t)^{1-\alpha}K(t)^\alpha - c(t)L \quad (10)\\
\dot{A}(t) &= (\sigma-1)\eta u(t)LA(t) \quad (11)\\
\dot{E}(t) &= z(t)^{(1+\beta)}(1-u(t)L)^{1-\alpha}A(t)^{1-\alpha}K(t)^\alpha - \theta E(t) \quad (12)\\
\lim_{t\to\infty} e^{-\rho t}K(t) &= 0 \quad (13)\\
\lim_{t\to\infty} e^{-\rho t}A(t) &= 0 \quad (14)\\
\lim_{t\to\infty} e^{-\rho t}E(t) &= 0. \quad (15)
\end{aligned}
$$

The r.h.s. of the capital accumulation equation (eq. (10)) follows from the equilibrium solution that the demand for each intermediate input adjusted for quality is equalized, or $\frac{x(i,t)}{A(i,t)} = \frac{x(t)}{A(t)} \equiv \frac{K(t)}{A(t)}$ (this will be derived later as part of the decentralized equilibrium in the next section), using the relationship in eq. (3) and the fact that $A(t) = \int_0^1 A(i,t)di$. The control variables for the planner's problem are consumption per capita, pollution intensity of output and the fraction of labor allocated to R and D, which are denoted by $c(t) \equiv C(t)/L$, $z(t)$ and $u(t)$, respectively. The state variables are aggregate capital stock (K(t)), state of knowledge (A(t)), and stock of environmental pollution (E(t)). We let λ_K, λ_A and λ_E denote the co-state variables associated with the three equations of motion in (10), (11), and (12). Finally, the last three constraints are the non-ponzi game conditions that determine the behavior of the three state variables in the terminal period of the analysis.

The current value Hamiltonian is defined as

$$
\begin{aligned}
H &= \left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] + \lambda_K\left(z(t)[(1-u(t))LA(t)]^{1-\alpha}K(t)^\alpha - c(t)L\right) + \\
&\quad \lambda_A(\sigma-1)\eta u(t)LA(t) - \lambda_E\left(z(t)^{(1+\beta)}((1-u(t))L)^{1-\alpha}A(t)^{1-\alpha}K(t)^\alpha - \theta E(t)\right). \quad (16)
\end{aligned}
$$

Then, the first-order conditions are

$$\frac{\partial H}{\partial c} = 0 \Leftrightarrow \quad c(t)^{-\epsilon} = \lambda_K L \tag{17}$$

$$\Leftrightarrow \quad \frac{\dot{c}}{c} \equiv g_c = -\frac{\dot{\lambda_K}}{\lambda_K}\frac{1}{\epsilon} \quad \text{(since } \dot{L} = 0 \text{ by assumption).} \tag{18}$$

$$\frac{\partial H}{\partial z} = 0 \Leftrightarrow \quad \lambda_K((1-u)LA)^{1-\alpha}K^{\alpha} - \lambda_E(1+\beta)z^{\beta}((1-u)LA)^{1-\alpha}K^{\alpha}$$

$$\Leftrightarrow \quad z^{\beta} = \tfrac{\lambda_K}{\lambda_E(1+\beta)} \text{ or } z = \left(\tfrac{\lambda_K}{\lambda_E(1+\beta)}\right)^{\frac{1}{\beta}}. \tag{19}$$

$$\frac{\partial H}{\partial u} = 0 \Leftrightarrow \quad -\lambda_K(1-\alpha)z(1-u)^{-\alpha}(LA)^{1-\alpha}K^{\alpha} + \lambda_E(1-\alpha)z^{(1+\beta)}(1-u)^{-\alpha}(LA)^{1-\alpha}K^{\alpha}$$

$$= \lambda_A(\sigma-1)\eta LA \tag{20}$$

$$\Leftrightarrow \quad (1-\alpha)\tfrac{Y}{(1-u)}\left(\lambda_K - \lambda_E z^{\beta}\right) = \lambda_A(\sigma-1)\eta LA \tag{21}$$

$$\Leftrightarrow \quad (1-\alpha)\tfrac{Y}{(1-u)}\lambda_K\tfrac{\beta}{(1+\beta)} = \lambda_A(\sigma-1)\eta LA, \tag{22}$$

where the simplification in the l.h.s. follows from the definition of Y and by using (19).

Next, the Euler equations are derived to be

$$\frac{\partial H}{\partial K} = \lambda_K\rho - \dot{\lambda_K} \Leftrightarrow \quad \alpha z((1-u)LA)^{1-\alpha}K^{\alpha-1}\left(\lambda_K - \lambda_E z^{\beta}\right)$$

$$= \lambda_K\rho - \dot{\lambda_K} \tag{23}$$

$$\Leftrightarrow \quad \alpha.\tfrac{Y}{K}\lambda_K\tfrac{\beta}{(1+\beta)} = \lambda_K\rho - \dot{\lambda_K}$$

$$\Leftrightarrow \quad \tfrac{\dot{\lambda_K}}{\lambda_K} = \rho - \alpha\tfrac{Y}{K}\tfrac{\beta}{(1+\beta)} \quad \text{(from (19))} \tag{24}$$

$$\frac{\partial H}{\partial A} = \lambda_A\rho - \dot{\lambda_A} \Leftrightarrow \quad (1-\alpha)z((1-u)L)^{1-\alpha}A^{-\alpha}K^{\alpha}\left(\lambda_K - \lambda_E z^{\beta}\right) + \lambda_A(\sigma-1)\eta uL$$

$$= \lambda_A\rho - \dot{\lambda_A} \tag{25}$$

$$\Leftrightarrow \quad (1-\alpha)z((1-u)L)^{1-\alpha}A^{-\alpha}K^{\alpha}\lambda_K\tfrac{\beta}{(1+\beta)} + \lambda_A(\sigma-1)\eta uL$$

$$= \lambda_A\rho - \dot{\lambda_A} \quad \text{(again from (19))} \tag{26}$$

$$\Leftrightarrow \quad \tfrac{\dot{\lambda_A}}{\lambda_A} = \rho - (\sigma-1)\eta L. \tag{27}$$

The last expression follows from the substitution of (22) into the l.h.s. of (26).

Further,

$$\frac{\partial H}{\partial E} = -(\lambda_E\rho - \dot{\lambda_E}) \Leftrightarrow \quad -\tfrac{E^{\delta}}{L} + \lambda_E\theta = -(\lambda_E\rho - \lambda_E) \tag{28}$$

$$\Leftrightarrow \quad \tfrac{\dot{\lambda_E}}{\lambda_E} = \rho + \theta - \tfrac{E^{\delta}}{\lambda_E L}. \tag{29}$$

Finally, the transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K K(t) = 0 \quad (30)$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_A A(t) = 0 \quad (31)$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_E E(t) = 0 \quad (32)$$

3.2 Steady state analysis

We focus on the behavior of the key variables along the balanced growth path, a detailed derivation of which is presented in Appendix B.

The substitution of (10) or the K-accumulation equation, into the r.h.s. of (18) entails

$$g_c = \frac{1}{\epsilon}\left[\alpha\frac{\beta}{(1+\beta)}\frac{Y}{K} - \rho\right] \Leftrightarrow g_c = \frac{1}{\epsilon}\left[\alpha\frac{\beta}{(1+\beta)}\left(g_K + \frac{C}{K}\right) - \rho\right]. \quad (33)$$

Furthermore, we get that

$$g_c = g_K = g_Y. \quad (34)$$

Proposition 1: *From (33) and the constancy of $\frac{Y}{K}$, it is possible for consumption to experience a positive rate of growth, despite a decline in pollution intensity, z, in the long-run. Further, the socially optimal growth rates of capital and output are the same as that for consumption per capita.* The first of the above follows from the fact that innovations allow both Y and K to grow at the same time. Mathematically, this is evident from the fact that in this case we have $\frac{Y}{K} = z\left(\frac{(1-u)LA}{K}\right)^{1-\alpha}$, and the fact that a sufficient growth in A will permit constancy of $\frac{Y}{K}$, notwithstanding the decline in z over time. (That z will decline along the balanced growth path, and the parametric configuration that guarantees the sustainability of the growth process are derived later in this section). The equality in the growth rates of consumption per capita, capital and output in steady state is proved in Appendix B.

By solving the remaining equations of the model the dynamics of knowledge creation, pollution flow and pollution stock variables are also characterized (see Appendix B for details). Further, u is found to be constant in steady state. Hence, we have the following equations that describe the behavior of these variables along the socially optimal balanced growth path.

$$\frac{\dot{u}}{u} \equiv g_u = 0 \tag{35}$$

$$\frac{\dot{A}}{A} \equiv g_A = \left[1 + \frac{1}{\beta(1-\alpha)}\left(\frac{\epsilon+\delta}{1+\delta}\right)\right] g_K. \tag{36}$$

$$\frac{\dot{E}}{E} \equiv g_E = g_K - \left(\frac{\epsilon+\delta}{1+\delta}\right) g_K \text{ or } g_E = -\left(\frac{\epsilon-1}{1+\delta}\right) g_K \tag{37}$$

$$\frac{\dot{z}}{z} \equiv g_z = -\frac{1}{\beta}\left(\frac{\epsilon+\delta}{1+\delta}\right) g_K \tag{38}$$

We next derive an explicit solution for $g_c = g_Y = g_K$, which does indeed exist. It is derived in Appendix B that

$$g_K \equiv g_c \equiv g_Y = \frac{(\sigma-1)\eta L - \rho}{\epsilon + \frac{1}{\beta(1-\alpha)}\left(\frac{\epsilon+\delta}{1+\delta}\right)}, \tag{39}$$

which is the closed-form solution for the growth rates of consumption per capita, capital stock and output of the final good along the balanced growth path for the planner's problem. This yields the following specific conditions (in terms of the parametric restrictions) which ensure long-run sustainability of growth.

$$(\sigma-1)\eta L - \rho > 0 \quad \text{(from (39))} \tag{40}$$

$$\epsilon - 1 > 0 \quad \text{(from (B8))} \tag{41}$$

The condition in (40) is required for consumption per capita, capital stock and output to grow indefinitely. The intuition is that sustainability is guaranteed if the size and rate of arrival of innovations are high enough, or that R and D is productive enough (relative to the rate of time preference). The restriction in (41) ensures a long-run decline in the pollution stock, E, over time.

Proposition 2: *The above parametric restrictions ensure that in the socially optimal long-run equilibrium situation, (i) consumption per capita (c), capital stock (K) and output of the final good (Y) grow without bound, (ii) knowledge derived from vertical innovations (or A) also grows in an unbounded fashion, and (iii) pollution intensity (z) and pollution stock (E) fall.*

The result in (i) follows from (39)) and the restriction in (40), while (ii) is ensured by (36). Finally, the outcomes in (iii) are implied by (38) and (37) combined with (41).

Observation 1: *Along the balanced growth path of the planner's equilibrium,* $g_A > g_K$.

This is clear from (36). The intuition is that, along the balanced growth path, knowledge needs to be accumulated at a rate faster than physical capital accumulation because, to ensure sustainable growth, it needs to counter two effects: diminishing returns to capital and the lowering of productivity due to stricter environmental restriction implied by $g_z < 0$ (see eq. (38)).

Observation 2: *At the planner's equilibrium, it is indeed the case that* $r > 0$ *and* $0 < u < 1$.
We have $g_c = \frac{r-\rho}{\epsilon} > 0$ (from (40)), together with $\rho > 0$ and $\epsilon > 0$ implies that $r > 0$. Further, $g_c = g_Y = \frac{1}{1-\alpha} g_z + g_A$ (from (B12) in Appendix B), combined with (5) implies

$$\begin{aligned} g_c &= \frac{1}{1-\alpha} g_z + (\sigma-1)\eta u L \\ \Leftrightarrow g_c &= \frac{1}{1-\alpha}\left(-\frac{1}{\beta}\left(\frac{\epsilon+\delta}{1+\delta}\right)\right) g_c + (\sigma-1)\eta u L \\ \Rightarrow u &= \frac{1+\frac{1}{(1-\alpha)\beta}\left(\frac{\epsilon+\delta}{1+\delta}\right) g_c}{(\sigma-1)\eta u L} > 0, \end{aligned} \tag{42}$$

which utilizes the expression for g_z from (B6). Next, substituting for g_c from (39) it is easy to show that $0 < u < 1$.

Thus, the optimal path for the planner's economy is fully characterized. We now describe and characterize the optimal outcomes for the unregulated decentralized economy.

4 The unregulated market equilibrium

4.1 Final good producers

As stated earlier, the markets for both good Y and factor L are assumed to be competitive, while the quantity of each intermediate input, $x(i,t)$, is bought at the monopoly price. In each period, t, the final good producers solve the following optimization problem with respect to their choice of labor and the range of intermediate inputs or $x(i,t)$s.

$$\begin{aligned} Max_{\{(1-u)L, x(i,t)_{i=0}^{1}\}} \Pi_Y(t) = \quad & z(t)((1-u(t))L)^{1-\alpha} \textstyle\int_0^1 A(i,t)^{1-\alpha} x(i,t)^{\alpha} di - \\ & w(t)(1-u(t))L - \textstyle\int_0^1 p(i,t)x(i,t)di, \end{aligned} \tag{43}$$

where, recall that the price of the final good is normalized to unity, $w(t)$ denotes unit wage rate and $p(i,t)$ is the unit monopoly price of the ith intermediate good. The first-order conditions imply that

$$w(t) = (1-\alpha)z(t)((1-u(t))L)^{-\alpha} \int_0^1 A(i,t)^{1-\alpha} x(i,t)^{\alpha} di \equiv (1-\alpha)\frac{Y(t)}{(1-u(t))L} \tag{44}$$

$$p(i,t) = \alpha z(t)((1-u(t))L)^{1-\alpha} A(i,t)^{1-\alpha} x(i,t)^{\alpha} \equiv \alpha \frac{Y(t)}{x(i,t)}. \tag{45}$$

The last expression yields the demand for each intermediate good as

$$x(i,t) = \left(\frac{\alpha z}{p(i,t)}\right)^{\frac{1}{1-\alpha}} (1-u)LA(i,t). \tag{46}$$

Thus, the demand for higher quality or more productive intermediate good is found to be higher.

4.2 Monopolist in the ith intermediate commodity sector

The unregulated monopolist in the ith intermediate good sector also maximizes profits with respect to his/ her choice of capital, which is the only input needed to produce $x(i,t)$. That is,

$$\begin{aligned} Max_{\{x(i,t)\}}\Pi_x(i,t) &= p(i,t)x(i,t) - r(t)K(i,t) \\ &= \alpha z(t)((1-u(t))L)^{1-\alpha}x(i,t)^{\alpha-1}x(i,t) - r(t)x(i,t), \end{aligned} \tag{47}$$

where the expression in the r.h.s. derives from the substitution of solution to $p(i,t)$ (from (45)) and $x(i,t) = K(i,t)$. $r(t)$ is the return on investment or price per unit capital. The first-order condition with respect to $x(i,t)$ determines its equilibrium level, which is

$$\tilde{x}(i,t) = \left(\frac{\alpha^2 z(t)}{r(t)}\right)^{\frac{1}{1-\alpha}} (1-u)LA(i,t), \tag{48}$$

where superscript ˜ refers to solutions for variables at the unregulated market outcome and whose substitution into the price equation in (45), yields the solution to equilibrium price as

$$\tilde{p}(i,t) = \alpha z(t)((1-u(t))L)^{1-\alpha}A(i,t)^{1-\alpha}\left(\frac{\alpha^2 z(t)}{r(t)}\right)^{\frac{\alpha-1}{1-\alpha}}((1-u(t))L)^{\alpha-1}A(i,t)^{\alpha-1} \equiv \frac{r(t)}{\alpha}. \tag{49}$$

This is the monopoly price charged as a markup over marginal cost. Note that, being independent of i, it is constant across all the intermediate goods. From (46) this leads to the quality/ productivity adjusted equilibrium quantity of each i being the same, or

$$\frac{\tilde{x}(i,t)}{A(i,t)} \equiv \frac{\tilde{x}(t)}{A(t)} = \frac{\tilde{K}(t)}{A(t)}.$$

This property is used in deriving the r.h.s. of (10) in the social planner's problem in the previous section, as well as the last expression in the r.h.s. of both (44) and (45).

Accordingly, the net profit of the ith monopolist in equilibrium will be expressed as

$$\tilde{\Pi}_x(i,t) = (\tilde{p}(i,t) - r(t))\,\tilde{x}(i,t) \equiv \left(\frac{r(t)}{\alpha} - r(t)\right)\tilde{x}(i,t) \equiv \frac{(1-\alpha)}{\alpha}r(t)\tilde{x}(i,t) \tag{50}$$

$$= (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}r(t)^{-\frac{\alpha}{1-\alpha}}z(t)^{\frac{1}{1-\alpha}}(1-u(t))LA(i,t). \tag{51}$$

More specifically, the monopolist in the ith intermediate good sector will always employ the best-available innovation to produce $x(i,t)$, implying that the associated profits will be

$$\tilde{\Pi}_x(t) = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} r(t)^{-\frac{\alpha}{1-\alpha}} z(t)^{\frac{1}{1-\alpha}} (1-u(t))L\bar{A}(t). \tag{52}$$

Note that,

Observation 3: *Equilibrium monopoly profits are declining in $r(t)$ and rising in $z(t)$.*

We next determine the equilibrium in the R and D sector.

4.3 Research arbitrage

The equilibrium in the R and D sector is determined by trading off the expected marginal benefit of the new innovation to the monopolist in the ith intermediate good sector (capturing his willingness-to-pay), against the marginal cost (of labor) of producing the new innovation. That is

$$\eta V(t) = w(t), \tag{53}$$

where

$$\begin{aligned} V(t) &= \int_t^\infty \Pi_x(i,\tau;\overline{A}(t)) e^{-\int_t^\tau r(s)ds} e^{-\int_t^\tau \eta u(s)Lds} d\tau \\ &= \int_t^\infty \frac{1-\alpha}{\alpha} r(\tau) x(i,\tau;\overline{A}(t)) e^{-\int_t^\tau r(s)ds} e^{-\int_t^\tau \eta u(s)Lds} d\tau. \end{aligned} \tag{54}$$

Here, the assumption is that monopolist's profits are discounted by two factors, one is the rate of return on alternative investment and the other is the probability of survival as arrival of the new innovation makes the existing patent obsolete. Combining this with the expression for $w(t)$ in (44) yields the research arbitrage equation to be:

$$\eta \int_t^\infty \frac{1-\alpha}{\alpha} r(\tau) x(i,\tau;\overline{A}(t)) e^{-\int_t^\tau r(s)ds} e^{-\int_t^\tau \eta u(s)Lds} d\tau = (1-\alpha) z(t) \left((1-u(t))L\right)^{-\alpha} \int_0^1 A(i,t)^{1-\alpha} x(i,t)^\alpha di$$

$$\Leftrightarrow \eta \int_t^\infty \frac{1-\alpha}{\alpha} r x(i,\tau;\overline{A}(t)) e^{-(r+\eta uL)(\tau-t)} d\tau = (1-\alpha) z(t) \left((1-u)L\right)^{-\alpha} \int_0^1 A(i,t)^{1-\alpha} x(i,t)^\alpha di.$$

$$\Leftrightarrow \eta\alpha(1-u)L \frac{\overline{A}(t)}{A(t)} z^{-\frac{1}{1-\alpha}} \int_t^\infty z(\tau)^{\frac{1}{1-\alpha}} e^{-(r+\eta uL)(\tau-t)} d\tau = 1 \tag{55}$$

$$\Leftrightarrow \eta\alpha(1-u)L\sigma \int_t^\infty e^{-(r+\eta uL-\frac{1}{1-\alpha}\tilde{g}_z)(\tau-t)} d\tau = 1 \tag{56}$$

The last two equations use the fact that r and u are constant in steady state. The condition in (56) can be simplified to

$$r + \eta uL - \frac{1}{1-\alpha}\tilde{g}_z = \eta\alpha\sigma(1-u)L, \tag{57}$$

which constitutes one equation that expresses r and u as a function of $z(t)$.[1] The other equation in r and u, again as a function of $z(t)$, will be obtained from the consumer' optimization exercise, that is, equation (71) in the next section. Together the two will solve for optimal levels, $\tilde{r}$ and $\tilde{u}$, at the unregulated market equilibrium.

4.4 Consumer's optimization

The representative consumer in this economy solves

$$\begin{aligned} Max\ U\left(c(0), \frac{E(0)}{L}\right) &= \int_0^\infty e^{-\rho t}\left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] dt \\ \text{s.t.} \quad \dot{a}(t) &= w(t) + r(t)a(t) - c(t) \qquad (58) \\ \dot{E}(t) &= Y(t)z(t)^{\beta} - \theta E(t) \qquad (59) \\ \lim_{t\to\infty} e^{\int_0^t r(\tau)d\tau} a(t) &= 0 \qquad (60) \\ \lim_{t\to\infty} e^{-\rho t} E(t) &= 0, \qquad (61) \end{aligned}$$

where $c(t)$, $\frac{E(t)}{L}$, ρ, ϵ, δ, $w(t)$ and $r(t)$ are as defined earlier, and $a(t) = \frac{Assets(t)}{L}$. Eq. (58) constitutes the wealth constraint of the consumer, (59) is the equation of motion of the pollution stock, E, and (60) and (61) are the non-ponzi game conditions. In equilibrium, $\frac{Assets}{L} = \frac{K}{L}$, as at the aggregate level there is no net lending and borrowing amongst consumers. Consumers optimize with respect to the consumption level, $c(t)$, as environmental quality is unregulated and is not determined by them. Thus, the environmental externality is not internalized in consumer's maximization of utility. The state variables are assets per capita, or $a(t)$, and environmental quality, $E(t)$, with λ_a and λ_e denoting the co-state variables associated with the assets dynamics equation in (58).

The current value Hamiltonian for the above dynamic optimization problem will be

$$H = \left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] + \lambda_a\left(w(t) + r(t)a(t) - c(t)\right) - \lambda_e\left(Y(t)z(t)^{\beta} - \theta E(t)\right).$$

The first-order condition for optimization will be

$$\begin{aligned} \frac{\partial H}{\partial c} &= 0 \Leftrightarrow c(t)^{-\epsilon} = \lambda_a \\ &\Rightarrow \frac{\dot{c}}{c} = -\frac{1}{\epsilon}\frac{\dot{\lambda}_a}{\lambda_a}. \qquad (62) \end{aligned}$$

Next, the Euler equations will be

$$\begin{aligned} \frac{\partial H}{\partial a} &= \lambda_a \rho - \dot{\lambda}_a \\ \Leftrightarrow \lambda_a r = \lambda_a \rho - \dot{\lambda}_a \quad \Leftrightarrow \quad \frac{\dot{\lambda}_a}{\lambda_a} &= \rho - r, \qquad (63) \end{aligned}$$

[1]The solution to the equation utilizes the Leibniz's rule

and

$$\begin{aligned} \frac{\partial H}{\partial E} &= -(\lambda_e \rho - \dot{\lambda}_e) \\ -\frac{E^\delta}{L} + \lambda_e \theta = -(\lambda_e \rho - \dot{\lambda}_e) &\Leftrightarrow \frac{\dot{\lambda}_e}{\lambda_e} = \rho + \theta - \frac{E^\delta}{L\lambda_e}. \end{aligned} \tag{64}$$

4.5 Growth rates in steady state

The substitution of (63) into (62) entails the familiar consumption growth equation

$$\frac{\dot{c}}{c} \equiv \tilde{g}_c = \frac{r - \rho}{\epsilon}. \tag{65}$$

Note that, from (65), since the steady state consumption per capita will grow at a constant rate, we will have to have r to be constant as well.

Next, eq. (48) implies

$$\begin{aligned} \int_0^1 \tilde{x}(i,t)di &= \left(\frac{\alpha^2 z(t)}{r(t)}\right)^{\frac{1}{1-\alpha}} (1-u(t))L \int_0^1 A(i,t)di \\ \Leftrightarrow x(t)(\equiv K(t)) &= \left(\frac{\alpha^2 z(t)}{r(t)}\right)^{\frac{1}{1-\alpha}} (1-u(t))LA(t), \end{aligned} \tag{66}$$

The capital accumulation eq. (10), implies

$$\begin{aligned} \frac{\dot{K}}{K} &= \frac{Y}{K} - \frac{cL}{K} \\ \Leftrightarrow \frac{\dot{K}}{K} &= z((1-u)L)^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha-1} - \frac{cL}{K} \\ \Leftrightarrow \frac{\dot{K}}{K} &= z((1-u)L)^{1-\alpha}\left[\left(\frac{\alpha^2 z)}{r}\right)^{\frac{1}{1-\alpha}}(1-u))L\right]^{\alpha-1} - \frac{cL}{K} \quad \text{(from (66))} \qquad (67) \\ \Leftrightarrow \frac{\dot{K}}{K} &= \frac{r}{\alpha^2} - \frac{cL}{K}. \qquad (68) \end{aligned}$$

Along the balanced growth path, the l.h.s. of eq. (68) is constant. Further, in the r.h.s., $\frac{Y}{K} = \frac{r}{\alpha^2}$ is also constant, since r is constant in steady state. This implies that $\frac{cL}{K}$ is also constant, which yields that in the decentralized equilibrium,

$$\tilde{g}_c = \tilde{g}_K = \tilde{g}_Y, \tag{69}$$

where again the variables with superscript ˜ refer to those at the unregulated decentralized equilibrium.

Further, from the substitution of (48) into (1) or the production function in the final good sector, we get

$$
\begin{aligned}
Y(t) &= \alpha^{\frac{2\alpha}{1-\alpha}}(1-u)Lr^{-\frac{\alpha}{1-\alpha}}z(t)^{\frac{1}{1-\alpha}}A(t) \\
\Rightarrow \frac{\dot{Y}}{Y} &= \frac{1}{1-\alpha}\frac{\dot{z}}{z}+\frac{\dot{A}}{A} \quad \text{(this uses the fact that } r \text{ and } u \text{ are constant in steady state)} \\
\Leftrightarrow \tilde{g}_Y \equiv \tilde{g}_c &= \frac{1}{1-\alpha}\tilde{g}_z+(\sigma-1)\eta uL \quad \text{(which uses eq.(11))}. \qquad (70)
\end{aligned}
$$

Combining the last equation with (65), we have

$$
\frac{r-\rho}{\epsilon} = \frac{1}{1-\alpha}\tilde{g}_z+(\sigma-1)\eta uL. \qquad (71)
$$

This constitutes the second equation that defines r and u as function of $z(t)$. Together with eq. (57), this is solved to provide the following solutions to r and u:

$$
\begin{aligned}
\tilde{r} &= \frac{1}{(1-\alpha)}\tilde{g}_z\left[\frac{\epsilon(\sigma-1)+\epsilon(\alpha\sigma+1)}{\epsilon(\sigma-1)+(\alpha\sigma+1)}\right]+\frac{\rho(\alpha\sigma+1)+\eta\alpha\sigma L(\sigma-1)}{\epsilon(\sigma-1)+(\alpha\sigma+1)}, \qquad (72) \\
\tilde{u} &= -\frac{1}{(1-\alpha)\eta L}\tilde{g}_z\left[\frac{\epsilon-1}{\epsilon(\sigma-1)+(\alpha\sigma+1)}\right]+\frac{1}{\eta L}\left[\frac{\eta\alpha\sigma L-\rho}{\epsilon(\sigma-1)+(\alpha\sigma+1)}\right], \qquad (73)
\end{aligned}
$$

details of which can be found in Appendix C.

Next, from the consumption growth path defined by (65), we get that

$$
\tilde{g}_c = \tilde{g}_Y = \tilde{g}_K = \frac{1}{\epsilon(\sigma-1)+(\alpha\sigma+1)}\left[\sigma\frac{(1+\alpha)}{(1-\alpha)}\tilde{g}_z+(\sigma-1)(\eta\alpha\sigma L-\rho)\right], \qquad (74)
$$

and, from (59), along the balanced growth path,

$$
\tilde{g}_E = \frac{1}{\epsilon(\sigma-1)+(\alpha\sigma+1)}\left[\left(\sigma\frac{(1+\alpha)}{(1-\alpha)}+\beta\right)\tilde{g}_z+(\sigma-1)(\eta\alpha\sigma L-\rho)\right]. \qquad (75)
$$

Detailed derivations of the steady state behavior of variables are provided in Appendix C. Notably,

Observation 4: *At the unregulated market equilibrium, since the final good producers have no incentive to reduce the pollution intensity,* $\tilde{g}_z \geq 0$. Further,

Observation 5: *If* $\tilde{g}_z = 0$, *from (72) and (73), it is easy to show that* $\tilde{r} > 0$ *and* $0 < \tilde{u} < 1$. *But, when* $\tilde{g}_z > 0$, *while* $\tilde{r} > 0$ *without any restrictions,* $\tilde{u} > 0$ *implies* $\tilde{g}_z < \frac{(\eta\alpha\sigma L-\rho)(1-\alpha)}{\epsilon-1}$, *and* $\tilde{u} < 1$ *implies* $\tilde{g}_z > \frac{(\eta\alpha\sigma L-\rho)-\eta L(\epsilon(\sigma-1)+(\alpha\sigma+1))(1-\alpha)}{\epsilon-1}$. *These constitute the parametric restrictions on* $\tilde{g}_z$.

That $\tilde{g}_z > 0$ has important implications for the sustainability of the growth process.

Proposition 3: *At the unregulated market equilibrium, a trade-off arises between long run economic growth and environmental sustainability.*

This is obvious from the expressions in (74) and (75). From (74), a sufficient condition for growth to be sustainable, in the absence of any environmental concerns, is that $g_z \geq 0$ and that $(\eta\alpha\sigma L - \rho) > 0$. The former implies that z be non-decreasing along the steady-state, while the latter means that the size and rate of arrival of new innovations be high enough. From Observation 4, it is indeed the case that $g_z \geq 0$. However, combined with (75), this implies that a zero or positive rate of growth of z, which entails $\tilde{g}_E > 0$, implies that at some $t > t(0)$, environmental stock will deteriorate to a point such that $E > \bar{E}$, which rules out production. This highlights the conflict between economic growth and environmental deterioration.

We now turn to exploring the possibility of implementing the social optimum as a decentralized equilibrium by an appropriate choice of public policies.

5 The optimal public policy

The market economy described above is beset with three distortions - pollution externality, market power in the intermediate goods sectors, and knowledge spillovers in R and D. These can be corrected by introducing three distinct policy tools, targeting each of the market distortions directly, thus constituting the first-best policy package. These are:

1. an emissions tax, t_P, for the internalization of pollution damage,
2. an ad valorem subsidy on capital cost, s_K, to correct for the monopolists' market power in the intermediate goods sectors, and
3. a subsidy on R and D cost, s_A, to internalize the positive knowledge spillover effects.

In all other respects the structure of the economy remains the same as in the last section. We start with the production sectors.

5.1 Final good producers

The profit maximization problem of the final good producers is now modified to reflect the emissions tax on their pollution generation.

$$Max_{\{(1-u)L,x(i,t)_{i=0}^{1},z(t)\}}\Pi_Y(t) = z(t)((1-u(t))L)^{1-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^{\alpha}di - w(t)(1-u(t))L - \int_0^1 p(i,t)x(i,t)di - t_P P(t)$$
$$= z(t)((1-u(t))L)^{1-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^{\alpha}di - w(t)(1-u(t))L - \int_0^1 p(i,t)x(i,t)di - t_P z(t)^{(1+\beta)}((1-u(t))L)^{1-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^{\alpha}di. \quad (76)$$

Note that, given the imposition of the emissions tax, the producer now optimally chooses the pollution intensity of output, $z(t)$. The first-order conditions are equivalent to

$$w(t) = (1-\alpha)z(t)((1-u(t))L)^{-\alpha}\int_0^1 A(i,t)^{1-\alpha}x(i,t)^{\alpha}di\left(1-t_P z^{\beta}\right) \equiv (1-\alpha)\frac{Y(t)}{(1-u(t))L}\left(1-t_P z^{\beta}\right), \quad (77)$$
$$p(i,t) = \alpha z(t)((1-u(t))L)^{1-\alpha}A(i,t)^{1-\alpha}x(i,t)^{\alpha-1}\left(1-t_P z^{\beta}\right) \equiv \alpha\frac{Y(t)}{x(i,t)}\left(1-t_P z^{\beta}\right), \quad (78)$$
$$z^{\beta} = \frac{1}{(1+\beta)t_P} \Leftrightarrow t_P = \frac{1}{(1+\beta)z^{\beta}}. \quad (79)$$

The substitution of the last result can be used in the other two first-order conditions to simplify to

$$1-t_P z^{\beta} = \frac{\beta}{(1+\beta)}. \quad (80)$$

We next move to the intermediate good monopolist's optimization.

5.2 Monopolist in the ith intermediate good sector

In each of the intermediate good sectors, the monopolist's net profits are now modified to incorporate an ad valorem subsidy offered on the cost of capital that is used to produce the good. Thus, the intermediate good monopolist solves

$$Max_{\{x(i,t)\}}\Pi_x(i,t) = p(i,t)x(i,t) - (1-s_K)r(t)K(i,t)$$
$$= \alpha\frac{\beta}{(1+\beta)}z(t)((1-u(t))L)^{1-\alpha}x(i,t)^{\alpha-1}x(i,t) - (1-s_K)r(t)x(i,t), \quad (81)$$

where, as earlier, the first term in the r.h.s. follows by substituting for $p(i,t)$ from (78), and using (80) as well as $x(i,t) = K(i,t)$. The necessary condition for optimization of net profits yields the following expression for equilibrium $x(i,t)$, given the regulatory instruments:

$$x'(i,t) = \left(\frac{\beta}{(1+\beta)}.\frac{\alpha^2 z(t)}{(1-s_K)r(t)}\right)^{\frac{1}{1-\alpha}}(1-u(t))LA(i,t), \quad (82)$$

where variables with $'$ denote those at the optimally regulated market outcome. A comparison of the last expression under optimal regulation with that in the unregulated case, given by eq. (48), shows that, *ceteris paribus*, the equilibrium demand for input $x(i,t)$ will be lowered on account of the emissions tax, and propped up on account of the subsidy on capital. The substitution of the demand function into the equilibrium price expression in (78) entails,

$$p'(i,t) = \frac{(1-s_K)r(t)}{\alpha}, \tag{83}$$

implying a common equilibrium price across all the intermediate goods, i, $i \in [0,1]$. This is now lowered by the amount of the subsidy, s_K, on capital cost, thus correcting the monopoly price distortion.

Accordingly, the net profit of the monopolist at the regulated market equilibrium can be expressed as

$$\begin{aligned}
\Pi'_x(i,t) &= \big(p'(i,t) - (1-s_K)r(t)\big)\, x'(i,t) \equiv \left(\frac{(1-s_K)r(t)}{\alpha} - (1-s_K)r(t)\right) x'(i,t) \\
&\equiv \frac{1-\alpha}{\alpha}(1-s_K)r(t)x'(i,t) \qquad (84)\\
&= (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{\beta}{1+\beta}\right)^{\frac{1}{1-\alpha}} ((1-s_K)r(t))^{-\frac{\alpha}{1-\alpha}} z(t)^{\frac{1}{1-\alpha}}(1-u(t))L\bar{A}(t). \qquad (85)
\end{aligned}$$

Observation 6: *At the optimally regulated equilibrium, the monopolists profits are increasing in both – the subsidy on capital, s_K, and $z(t)$.*

The expression for equilibrium profits will now be used in the research arbitrage condition to derive the optimal level of R and D, given the public policies.

5.3 Research arbitrage

With the subsidy on R and D costs that would internalize the benefits of knowledge spillover, the research arbitrage condition is given by

$$\begin{aligned}
&\eta V(t) = (1-s_A)w(t) \\
\Leftrightarrow \eta \int_t^\infty (1-s_K)r(\tau)x'(i,\tau,\bar{A}(t))e^{-\int_t^\tau r(s)ds} e^{-\int_t^\tau \eta u(s)Lds} d\tau = \\
&(1-\alpha)(1-s_A)z(t)(1-u(t)L)^{-\alpha}\int_0^\infty A(i,t)^{1-\alpha}x(i,t)^\alpha di,
\end{aligned}$$

which, after assuming that both r and u are constant in time along the balanced growth path, and substituting for $x'(i,t,\bar{A}(t))$ from (82) can be expressed as

$$\eta \int_t^\infty (1-s_K) r \left(\frac{\beta}{(1+\beta)} . \frac{\alpha^2 z(\tau)}{(1-s_K) r}\right)^{\frac{1}{1-\alpha}} (1-u) L \bar{A}(t) e^{-(r+\eta u L)(\tau-t)} d\tau =$$

$$(1-\alpha)(1-s_A) z(t)((1-u)L)^{-\alpha} \int_0^\infty A(i,t)^{1-\alpha} \left(\frac{\beta}{(1+\beta)} . \frac{\alpha^2 z(t)}{(1-s_K) r}\right)^{\frac{\alpha}{1-\alpha}} ((1-u) L A(i,t))^\alpha$$

$$\Leftrightarrow \eta \frac{\alpha}{(1-s_A)} \frac{\beta}{(1+\beta)} \frac{\bar{A}(t)}{A(t)} (1-u) L z(t)^{-\frac{1}{1-\alpha}} \int_t^\infty z(\tau)^{\frac{1}{1-\alpha}} e^{-(r+\eta u L)(\tau-t)} d\tau = 1. \quad (86)$$

$$\Leftrightarrow \eta \frac{\alpha}{(1-s_A)} \frac{\beta}{(1+\beta)} (1-u) L \sigma \int_t^\infty e^{-(r+\eta u L - g_z')(\tau-t)} d\tau = 1. \quad (87)$$

This condition is analogous to the research arbitrage equation in (56) for the unregulated market situation, which can be solved similarly to yield

$$r + \eta u L - \frac{1}{(1-\alpha)} g_z' = \eta \frac{\alpha}{(1-s_A)} \frac{\beta}{1+\beta)} \sigma (1-u) L, \quad (88)$$

which provides one equation that expresses r and u as a function of $z(t)$. The other equation in r and u as a function of $z(t)$ is given by the optimal growth path in (104) in a later section. Together the two will solve for the steady state values of r and u under optimal regulation.

5.4 Utility maximization by the consumer

The representative consumer solves

$$Max\ U\left(c(0), \frac{E(0)}{L}\right) = \int_0^\infty e^{-\rho t} \left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta).L}\right] dt$$

s.t.

$$\dot{a}(t) = w(t) + r(t)a(t) - c(t) - \frac{T(t)}{L} \quad (89)$$

$$\dot{E}(t) = Y(t) z(t)^\beta - \theta E(t) \quad (90)$$

$$T(t) + t_P P(t) = s_K(t) r(t) K(t) + s_A w(t) u(t) \quad \forall t \quad (91)$$

$$\lim_{t\to\infty} e^{\int_0^t r(\tau) d\tau} a(t) = 0 \quad (92)$$

$$\lim_{t\to\infty} e^{-\rho t} E(t) = 0, \quad (93)$$

where $c(t)$, $\frac{E(t)}{L}$, ρ, ϵ, δ, $w(t)$, $r(t)$ and $a(t) = \frac{Assets(t)}{L}$ are as defined earlier, and so are the constraints in (89), (90), (92) and (93). In addition, $\frac{T(t)}{L}$ is the per capita lump sum tax imposed on the consumer, and also, there exists the budget constraint of the government as defined in (91).

The current value Hamiltonian for the above problem will be:

$$H = \left[\frac{c(t)^{1-\epsilon}}{1-\epsilon} - \frac{E(t)^{1+\delta}}{(1+\delta)L}\right] + \lambda_a \left(w(t) + r(t)a(t) - c(t) - \frac{T(t)}{L}\right) + \lambda_e \left(Y(t) z(t)^\beta - \theta E(t)\right).$$

The first-order condition for optimization will be derived by solving

$$\begin{aligned}\frac{\partial H}{\partial c} &= 0 \Leftrightarrow c(t)^{-\epsilon} = \lambda_a \\ \Rightarrow \quad \frac{\dot{c}}{c} &= -\frac{1}{\epsilon}\frac{\dot{\lambda}_a}{\lambda_a}.\end{aligned} \tag{94}$$

Furthermore, the Euler equations will be

$$\begin{aligned}\frac{\partial H}{\partial a} &= \lambda_a \rho - \dot{\lambda}_a \\ \Leftrightarrow \lambda_a r = \lambda_a \rho - \dot{\lambda}_a \quad \Leftrightarrow \quad -\frac{\dot{\lambda}_a}{\lambda_a} &= r - \rho,\end{aligned} \tag{95}$$

and

$$\begin{aligned}\frac{\partial H}{\partial E} &= -(\lambda_e \rho - \dot{\lambda}_e) \\ -\frac{E^\delta}{L} + \lambda_e \theta = -(\lambda_e \rho - \dot{\lambda}_e) \quad \Leftrightarrow \quad \frac{\dot{\lambda}_e}{\lambda_e} &= \rho + \theta - \frac{E^\delta}{L\lambda_e}.\end{aligned} \tag{96}$$

We also have the government budget balance condition being met in each period, which is

$$T(t) + t_P P(t) = s_K(t) r(t) K(t) + s_A w(t) u(t) \quad \forall t. \tag{97}$$

5.5 Growth rates in steady state

The substitution of (95) into (94) entails the familiar consumption growth equation to be

$$\frac{\dot{c}}{c} \equiv {g'}_c = \frac{r - \rho}{\epsilon}. \tag{98}$$

The growth rates denoted with superscript $'$ refer to those at the optimally regulated equilibrium. Further, by integrating both the sides of (84) with respect to i, we get

$$\begin{aligned}\int_0^1 x'(i,t)di &= \left(\frac{\beta}{(1+\beta)}\frac{\alpha^2 z(t)}{(1-s_K)r(t)}\right)^{\frac{1}{1-\alpha}}(1-u)L\int_0^1 A(i,t)di \\ \Rightarrow x'(t) \equiv K(t) &= \left(\frac{\beta}{(1+\beta)}\frac{\alpha^2 z(t)}{(1-s_K)r(t)}\right)^{\frac{1}{1-\alpha}}(1-u)LA(t) \\ \Rightarrow \frac{x'(t)}{A(t)} \equiv \frac{K(t)}{A(t)} &= \left(\frac{\beta}{(1+\beta)}\frac{\alpha^2 z(t)}{(1-s_K)r(t)}\right)^{\frac{1}{1-\alpha}}(1-u)L,\end{aligned}$$

whose substitution into the final good's production function yields

$$\begin{aligned}\frac{Y}{K} &= z((1-u)L)^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha-1} \\ &= z((1-u)L)^{1-\alpha}\left[\left(\frac{\alpha^2 z}{(1-s_K)r}\right)^{\frac{1}{1-\alpha}}(1-u)L\right]^{\alpha-1} = \frac{(1-s_K)r}{\alpha^2},\end{aligned} \tag{99}$$

which will be constant in steady state due to the constancy of r.

Hence, along the steady state

$$g'_Y = g'_K. \tag{100}$$

Notably, from the government budget balance condition in (97) together with (6),(77) and (79), one has

$$\begin{aligned} \frac{T}{K} + t_P z^{\beta}\frac{Y}{K} &= s_K r + s_A(1-\alpha)\frac{\beta}{(1+\beta)}\frac{1}{(1-u)L}\frac{Y}{K}uL \\ \Leftrightarrow \frac{T}{K} + \frac{1}{1+\beta}\frac{Y}{K} &= s_K r + s_A(1-\alpha)\frac{\beta}{(1+\beta)}\frac{u}{(1-u)}\frac{Y}{K}, \end{aligned}$$

which implies that the ratio $\frac{T}{K}$ will be constant along the steady state, a result that stems from the fact that, in steady state, we have $\frac{Y}{K}$ as constant (from (100)), and so are r, u, s_K and s_A. Thus,

$$g'_T = g'_K. \tag{101}$$

Furthermore, from the aggregate capital dynamics equation, we have

$$\begin{aligned} \frac{\dot{K}}{K} &= \frac{Y}{K} - \frac{cL}{K} - \frac{T}{K} \\ \Rightarrow g'_c &= g'_K. \end{aligned} \tag{102}$$

The last equality follows from the constancy of $\frac{\dot{K}}{K}$, $\frac{Y}{K}$ and $\frac{T}{K}$ along the steady state.

Hence, even at the optimally regulated equilibrium, the steady state rates of growth of variables will be such that

$$g'_c = g'_Y = g'_K = g'_T. \tag{103}$$

Next, analogous to the unregulated decentralized equilibrium, one can substitute the solution for $x'(i,t)$ (from (82)) into the final goods production function to get

$$\begin{aligned} Y(t) &= z(t)((1-u)L)^{1-\alpha}\int_0^1 A(i,t)\left(\frac{\beta}{(1+\beta)}\frac{\alpha^2 z(t)}{(1-s_K)r}\right)^{\frac{\alpha}{1-\alpha}}((1-u)LA(i,t))^{\alpha}\,di \\ \Rightarrow Y &= \alpha^{\frac{2\alpha}{1-\alpha}}\left(\frac{\beta}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}}(1-u)L((1-s_K)r)^{-\frac{\alpha}{1-\alpha}}z(t)^{\frac{1}{1-\alpha}}A(t) \\ \Rightarrow g'_Y \equiv g'_c &= \frac{r-\rho}{\epsilon} = \frac{1}{1-\alpha}g'_z + g_A \equiv \frac{1}{1-\alpha}g'_z + (\sigma-1)\eta uL, \end{aligned} \tag{104}$$

which utilizes the fact that both r and u will be constant along the balanced growth path.

Taking (88) and (104) and solving simultaneously for r and u yields the solutions to be:

$$r' = \frac{1}{1-\alpha}g'_z\left[\frac{\epsilon(\sigma-1)+\epsilon(\alpha'\beta'\sigma+1)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\right] + \left[\frac{\alpha'\beta'\eta\sigma L\epsilon(\sigma-1)+\rho(\alpha'\beta'\sigma+1)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\right]; \tag{105}$$

$$u' = -\frac{1}{(1-\alpha)\eta L}g'_z\left[\frac{\epsilon-1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\right] + \frac{1}{\eta L}\left[\frac{\alpha'\beta'\eta\sigma L-\rho}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\right], \tag{106}$$

where $\alpha' = \frac{\alpha}{1-s_A}$ and $\beta' = \frac{\beta}{1+\beta}$. More detailed derivations are provided in Appendix D.

Accordingly, the steady state rates of growth are worked out to be:

$$g'_c = g'_Y = g'_K = g'_T \quad = \quad \frac{1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\left[\sigma\frac{(\alpha'\beta'\sigma+1)}{(1-\alpha)}g'_z + (\sigma-1)(\alpha'\beta'\eta\sigma L-\rho)\right], \quad (107)$$

$$g'_E \quad = \quad \frac{1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)}\left[\left(\sigma\frac{(\alpha'\beta'\sigma+1)}{(1-\alpha)}+\beta\right)g'_z + (\sigma-1)(\alpha'\beta'\eta\sigma L-\rho)\right], \quad (108)$$

and from (D6), we have

$$g'_z = -\frac{1}{\beta}g_{t_P}. \quad (109)$$

This completes the characterization of steady-state growth rates for the key variables at the optimally regulated market equilibrium. Again, detailed derivations can be found in Appendix D. In what follows, we utilize the equivalence between growth rates at the planner's and the optimally regulated market equilibria to characterize the optimal policy package in terms of t_P, s_K and s_A.

5.6 Characterizing the optimal public policies

The optimal level of the individual policy tools are now derived so that these will implement the socially desirable outcome.

We first consider the subsidy on capital, s_K, which is the most straightforward to characterize. Note that, in the unregulated market regime, the monopolist's equilibrium mark-up in the ith intermediate good sector is captured by

$$\frac{\tilde{p}(i,t)-\tilde{r}}{\tilde{p}(i,t)} = \frac{\frac{\tilde{r}}{\alpha}-\tilde{r}}{\frac{\tilde{r}}{\alpha}} = 1-\alpha, \quad (110)$$

where the l.h.s. of (110) follows from the fact that the marginal cost producing $x(i,t)$ $\forall t$ is simply the marginal cost of capital, equal to r, and the r.h.s. uses the solution for monopoly equilibrium price in (49). At the regulated optimum, we need to design the capital subsidy so that the price-cost mark-up is now made to vanish. That is,

$$\frac{p'(i,t)-r'}{p'(i,t)} \quad \equiv \quad \frac{\frac{(1-s_K)r'}{\alpha}-r'}{\frac{(1-s_K)r'}{\alpha}} = 0 \quad \text{(uses the solution to equilibrium price in (83))}$$

$$\Leftrightarrow s_K \quad = \quad 1-\alpha. \quad (111)$$

This constitutes the optimum subsidy rate on physical capital. Thus,

Proposition 4: *A positive ad-valorem subsidy on capital, $s_K = 1-\alpha$ will correct for monopoly pricing in the intermediate goods sectors.*

A positive subsidy on capital is targeted at correcting for the market power of the intermediate good's monopolists. However, by raising the monopoly profits (by Observation 6), it also works toward stimulating innovation, thus raising the productivity of the final good sector.

Next, characterization of the pollution tax, t_P, relies on establishing equivalence between the growth rates at the social optimum and in the optimally regulated market equilibrium. From (D4) (in Appendix D) we have,

$$\begin{aligned} g'_E &= g'_Y + \beta g'_z \\ &= g'_c + \beta g'_z \\ &= g_c + \beta\left(-\frac{1}{\beta}\left(\frac{\epsilon+\delta}{1+\delta}\right)\right) g_c, \end{aligned}$$

which uses the growth rate equivalence and the solution for g_z in Appendix B. This implies

$$g'_E = -\left(\frac{\epsilon-1}{1+\delta}\right) g_c < 0 \text{ (from the restriction in (41))} \tag{112}$$

Further, re-expressing,

$$\begin{aligned} g'_E &= g'_c - g_{t_P} \text{ (from using (D4) and (D6))} \\ \Leftrightarrow -\left(\frac{\epsilon+\delta}{1+\delta}\right) g_c &= g_c - g_{t_P} \text{ (from using the growth rate equivalence and (112))} && (113) \\ \Rightarrow g_{t_P} &= \left(\frac{\epsilon+\delta}{1+\delta}\right) g_c && (114) \\ \Rightarrow t_P &= t_P(0) e^{\left(\frac{\epsilon+\delta}{1+\delta}\right) g_c t}, && (115) \end{aligned}$$

where $t_P(0)$ is the initial level of tax the economy starts out with. In view of the closed-form solution for g_c at the planner's equilibrium (defined by (39)), the solution to the optimal pollution tax depicts a constant rate of change over time along the steady state. Notably,

Proposition 5: *At the optimally regulated market equilibrium, a constant positive rate of growth of pollution tax along the steady state, implied by (115), will be consistent with long-run sustainable economic growth. Moreover, since $\epsilon > 1$, from (114) this rate of increase of optimal tax will be higher than the consumption or output growth rate.*

Finally, as for the optimal subsidy on R and D activity, s_A, from the solution to u' in (106), we get that

$$\begin{aligned} 1 - s_A &= \frac{(1-u')\alpha\beta'\eta\sigma}{\left(\frac{\epsilon-1}{1-\alpha}\right) g'_z + \rho + u'\eta L(\epsilon(\sigma-1)+1)} \\ \Rightarrow 1 - s_A &= \frac{\frac{\alpha\beta\eta\sigma L}{(1+\beta)}\left[1 - \frac{1}{(\sigma-1)\eta L}\left(1 + \frac{\epsilon+\delta}{\beta(1-\alpha)(1+\delta)}\right) g_c\right]}{\rho + \left[\frac{\sigma(\epsilon+\delta)+\beta(1-\alpha)(1+\delta)(\epsilon(\sigma-1)+1)}{\beta(1-\alpha)(1+\delta)(\sigma-1)}\right] g_c} > 0 && (116) \end{aligned}$$

after substituting for $u' = u(g_c)$ from (42) and $g_z' = g_z(g_c)$ from (38), and the balanced growth result, $g_K = g_c$ at the planner's equilibrium. Thus, $s_A < 1$. From what follows immediately, it is analytically difficult to unambiguously show that $s_A > 0$. Specifically, a sufficient condition for $s_A > 0$ will be that

$$\frac{[\sigma(\epsilon+\delta)(1+\beta+\alpha\beta)+\alpha\beta^2\sigma(1-\alpha)(1+\delta)+(1-\alpha)\beta(1+\beta)(1+\delta)(\epsilon(\sigma-1)+1)]}{(1-\alpha)\beta(1+\beta)(1+\delta)(\sigma-1)}g_c > \frac{\alpha\beta\eta\sigma L}{(1-\alpha)\beta(1+\beta)(1+\delta)} - \rho. \tag{117}$$

Thus, a high enough consumption growth rate, g_c, implied, in turn, by a productive enough R and D activity, will render a positive R and D subsidy at the optimally regulated equilibrium. However, a theoretical possibility of a tax on R and D ($s_A < 0$) cannot be ruled out if the R and D sector is not sufficiently productive.

Proposition 6: *At the optimally regulated decentralized equilibrium, $1 - s_A > 0$. Moreover, in general, $s_A > 0$, or it is optimal to provide a subsidy to R and D activity to internalize the positive knowledge spillover effect. However, a theoretical possibility of a tax on R and D cannot not ruled out, in case R and D is not sufficiently productive.*

A positive subsidy to the R and D sector is primarily targeted at internalizing the positive externality effect of knowledge spillover. However, similar to the capital subsidy, by spurring innovation, it raises the output of the final good sector. Thus,

Proposition 7: *At the optimally regulated equilibrium, a tightening environmental policy (a positive and rising t_P) impacts economic growth through depressing the output of the final good sector, reducing the marginal benefits from innovation by lowering the demand for intermediate goods, and, lowering the marginal cost of innovation by depressing the demand for labour in the final good sector. A positive subsidy to capital ($s_K > 0$) counters the depressing effect on the demand for intermediate goods, by raising the profits of the monopoly producers, thus enhancing the marginal value of innovations, while a positive subsidy to the R and D sector ($s_A > 0$) lowers the marginal cost of R and D. Both of these effects speed up innovation activity to offset the dampening effect of a stricter environmental policy on economic growth.*

This completes the discussion on the optimal policy tool-kit. Furthermore,

Observation 8: *Given the equivalence between the outcomes at the regulated equilibrium and the social optimum, $r' = r > 0$. Furthermore, $0 < u' = u < 1$.*

Proposition 8: *At the optimally regulated market equilibrium, the sustainability of growth is* *guaranteed by (i) the growth equivalence of per capita consumption (c), capital stock (K), and output (Y), with that at the social optimum, (ii) a declining pollution intensity, z, and (iii) a fall in the pollution stock, E.*

The growth equivalence in (i) above is ensured by a tightening pollution policy and a subsidy on R and D, described in Propositions 5 and 6, while (109) and (112) support (ii) and (iii) in Proposition 7 above.

Thus, the behaviour of the variables along the balanced growth path of the unregulated equilibrium is fully characterized.

6 Conclusions

The paper attempts to derive the optimal policy package to implement the social optimum of an Aghion-Howitt economy with environmental pollution. Our benchmark case is the planner's equilibrium that displays the possibility of long-run sustainable growth of output, consumption and capital stock, an unbounded growth of knowledge through vertical innovations, and a fall in both – the intensity and aggregate stock – of environmental pollution. By comparison, at the unregulated market equilibrium, a continued growth in pollution stock arrests the possibility of sustainable economic growth, thus highlighting the trade-off between growth and environmental protection. The first-best policy kit comprises a positive and rising tax on pollution, a subsidy on capital and a subsidy on innovations, when R and D is sufficiently productive. A possibility of a tax on R and D activity is not ruled out by our analysis. The rising pollution tax affects economic growth through depressing the output of the final good sector, reducing the value of innovations by lowering the demand for intermediate goods, and by reducing the cost of innovation through releasing labour from the final good sector. In general, both – the capital and the R and D subsidies – stimulate innovations and raise the productivity of the final good sector to offset the dampening effect of a declining pollution intensity on final sector output.

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Appendix A

The dynamic equation for the knowledge variable, A, is derived here.

Given $A(i,t) = \sigma A(i,t-1)$ and starting from time period t, the expected level of arrival of successive innovation at $t + \triangle t$ is given by

$$\begin{aligned}
E(A(i,t+\triangle t)) &= \sigma\eta u(t)LA(i,t)\triangle t + (1-\eta u(t)L)A(i,t)\triangle t \\
\Leftrightarrow E(A(i,t+\triangle t)) &= (\sigma-1)\eta u(t)LA(i,i)\triangle t + A(i,t)\triangle t \\
\Leftrightarrow \frac{E(A(i,t+\triangle t)) - A(i,t)\triangle t}{\triangle t} &= (\sigma-1)\eta u(t)LA(i,i) \\
\Leftrightarrow \frac{\dot{A}(i,t)}{A(i,t)} &= (\sigma-1)\eta u(t)L, \qquad \text{(A1)}
\end{aligned}$$

for $\triangle t$ small enough. The innovation process is assumed to flow in the same fashion for the leading edge technology, $\bar{A}(t)$, thus yielding the innovation flow equation (5).

In view of uniformity of R and D intensity across all the intermediate goods sectors, or, $u(i,t) = u(t)\forall i$, the average productivity index, $A(t)$ is found to be proportional to the leading edge productivity index, $\overline{A}(t)$. Specifically, $\frac{\overline{A}(t)}{A(t)} = \sigma$. The method of proof follows the one in Grimaud and Ricci (1999), and is provided below.

Let $G(\dot{},t)$ denote the cumulative distribution of the productivity parameter $A(i,t)\forall i \in [0,1]$ in steady state, and let $A(0)$ represent the leading edge technology at time t(0). Thus,

$$G(A(0),t(0)) = 1 \;\text{ such that }\; \frac{dG}{dt} = -\eta uLG(A(0),t),$$

which implies that at time $t(0)$, $A(0)$ is the frontier technology, or is at the top, and over time, it begins to slide down the quality ladder as new innovations arrive in other intermediate goods. That is, at some time $t > t(0)$,

$$\frac{\dot{G}}{G} = -\eta uL \Leftrightarrow G = e^{-\eta uL(t-t(0))}. \qquad \text{(A2)}$$

But, (A1) together with $A(t(0)) = A(0)$ imply that

$$\overline{A}(t) = A(0)e^{(\sigma-1)\eta uL(t-t(0))} \Leftrightarrow \frac{A(0)}{A(t)} = e^{-(\sigma-1)\eta uL(t-t(0))}. \qquad \text{(A3)}$$

From (A2) and (A3), we get

$$G^{\sigma-1} = \frac{A(0)}{\overline{A}(t)} \Leftrightarrow G = \left[\frac{A(0)}{\overline{A}(t)}\right]^{\frac{1}{\sigma-1}}.$$

Next, define $a(i,t) = \frac{A(i,t)}{\overline{A}(t)} \in (0,1]$ as the relative productivity parameter in relation to the frontier technology. As the past memory fades away, we have,

$$\begin{aligned} \tilde{G}(a) &= a^{\frac{1}{\sigma-1}} \\ \Rightarrow \tilde{g}(a) = \frac{d\tilde{G}(a)}{da} &= \frac{1}{\sigma-1} a^{\frac{1}{\sigma-1}-1}. \end{aligned}$$

Moreover,

$$\begin{aligned} A(t) &= \int_0^1 A(i,t)di \\ &= \overline{A}(t) \int_0^1 a(i,t)di \\ &= \overline{A}(t) \int_0^1 a\tilde{g}(a)da \\ &= \overline{A}(t) \int_0^1 a \frac{a^{\frac{1}{\sigma-1}-1}}{\sigma-1} da \\ &= \overline{A}(t) \left[\frac{a^{\frac{1}{\sigma-1}+1}}{\sigma-1+1} \right]_{a=0}^{a=1} \\ &= \overline{A}(t) \left[\frac{1}{\sigma} - 0 \right] \Leftrightarrow \frac{\overline{A}(t)}{A(t)} = \sigma. \end{aligned} \tag{A4}$$

Appendix B

This appendix derives the behavior of key variables in the steady state for the social planner's problem.

Take the equation of motion of K first, which yields

$$\frac{\dot{K}}{K} \equiv g_k = \frac{z((1-u)LA)^{1-\alpha}K^{\alpha}}{K} - \frac{C}{K} \quad \text{or} \quad \frac{z((1-u)LA)^{1-\alpha}K^{\alpha}}{K} = g_K + \frac{cL}{K}, \tag{B1}$$

whose substitution into the r.h.s. of (18) entails

$$g_c = \frac{1}{\epsilon}\left[\alpha \frac{\beta}{(1+\beta)} \frac{Y}{K} - \rho\right] \Leftrightarrow g_c = \frac{1}{\epsilon}\left[\alpha \frac{\beta}{(1+\beta)} \left(g_K + \frac{cL}{K}\right) - \rho\right]. \tag{B2}$$

Since, along the balanced growth path, g_c and g_K are constant and α, ϵ, β, and ρ are all parameters, in the above equation $\frac{cL}{K}$ will also be constant, implying $g_c = g_K$ as $\dot{L} = 0$. Furthermore, the last result together with (B1) yields that $\frac{Y}{K}$ is also constant. Hence, we have

$$g_c = g_K = g_Y. \tag{B3}$$

Taking (29) next, we have the l.h.s. constant in steady state. Further, both ρ and θ are parameters. Hence, $\frac{E^{\delta}}{\lambda_E L} \equiv \frac{E^{\delta} z^{\beta}(1+\beta)}{\lambda_K L}$ is also constant. The last expression follows from (19). Thus, we have

$$\delta\frac{\dot{E}}{E}-\frac{\dot{\lambda_K}}{\lambda_K}+\beta\frac{\dot{z}}{z}=0 \text{ or } \frac{\dot{E}}{E}=\frac{1}{\delta}\left[-\epsilon\frac{\dot{c}}{c}-\beta\frac{\dot{z}}{z}\right], \tag{B4}$$

which uses (18).

The other equation that relates to dynamics of E, namely eq. (12), yields that the ratio $\frac{Yz^\beta}{E}$ is constant in steady state, since both $\frac{\dot{E}}{E}$ and θ are constant. This implies that

$$\frac{\dot{E}}{E}=\frac{\dot{Y}}{Y}+\beta.\frac{\dot{z}}{z} \tag{B5}$$

Equating the r.h.s. of (B4) and (B5) and utilizing the equality of growth rates in (B3), we get

$$\begin{aligned}\frac{1}{\delta}\left[-\epsilon\frac{\dot{c}}{c}-\beta\frac{\dot{z}}{z}\right] &= g_c+\beta\tfrac{\dot{z}}{z}\\ \Leftrightarrow -\beta\left(\frac{1+\delta}{\delta}\right)g_z &= \left(\tfrac{\epsilon+\delta}{\delta}\right)g_c(\equiv g_K)\\ \Leftrightarrow g_z &= -\tfrac{1}{\beta}\left(\tfrac{\epsilon+\delta}{1+\delta}\right)g_K \qquad \text{(B6)}\end{aligned}$$

Substitution of (B6) into the r.h.s. of (B5) and again using growth equality in (B3), we have

$$\frac{\dot{E}}{E}=g_K-\left(\frac{\epsilon+\delta}{1+\delta}\right)g_K \text{ or } g_E=-\left(\frac{\epsilon-1}{1+\delta}\right)g_K \tag{B7}$$

As for the co-state variable λ_E, from (19), we can derive

$$\frac{\dot{\lambda_E}}{\lambda_E}=\tfrac{\dot{\lambda_K}}{\lambda_K}-\beta\tfrac{\dot{z}}{z}\Leftrightarrow g_{\lambda_E}=-\epsilon g_K-\left(\frac{\epsilon+\delta}{1+\delta}\right)g_K \text{ or } g_{\lambda_E}=-(1-\epsilon)\left(\frac{\delta}{1+\delta}\right)g_K.$$

The last expression in the r.h.s. follows from substituting with (18) and (B6).

Next, from (26), we get

$$\frac{\dot{\lambda_A}}{\lambda_A}=\rho-(\sigma-1)\eta uL-(1-\alpha)\frac{\lambda_K}{\lambda_A}\frac{\beta}{(1+\beta)}\frac{Y}{A}.$$

At the steady state, we will have $\frac{\dot{\lambda_A}}{\lambda_A}$ to be constant. Given ρ, σ, η and L as parameters, and u also constant in steady state (this will be derived later in the appendix), we have $(1-\alpha)(1-\alpha)\frac{\lambda_K}{\lambda_A}\frac{\beta}{(1+\beta)}\frac{Y}{A}$ also constant, such that

$$\frac{\dot{\lambda_K}}{\lambda_K}-\frac{\dot{\lambda_A}}{\lambda_A}+\frac{\dot{Y}}{Y}-\frac{\dot{A}}{A}=0 \tag{B8}$$

$$\Leftrightarrow \frac{\dot{\lambda_A}}{\lambda_A}=(1-\epsilon)g_K-g_A, \tag{B9}$$

which will be revisited later.

We take (20) next, which can be expressed as

$$\frac{z((1-u)LA)^{1-\alpha}K^\alpha}{K}=\frac{\lambda_A}{\lambda_K}(\sigma-1)\eta(1-u)L\frac{A}{K} \Leftrightarrow \frac{Y}{K}=\frac{\lambda_A}{\lambda_K}(\sigma-1)\eta(1-u)L\frac{A}{K}.$$

From our earlier derivations, we have the l.h.s. constant along the balanced growth path, implying constancy of the r.h.s. as well. Thus,

$$\begin{aligned} \frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_K}{\lambda_K} + \frac{\dot{A}}{A} - \frac{\dot{K}}{K} + \frac{\dot{(1-u)}}{(1-u)} &= 0, \\ \Leftrightarrow (1-\epsilon)g_K - \frac{\dot{A}}{A} + \epsilon g_K + \frac{\dot{A}}{A} - g_K + \frac{\dot{(1-u)}}{(1-u)} &= 0 \qquad \text{(B10)} \\ \Leftrightarrow \frac{\dot{(1-u)}}{(1-u)} &= 0 \text{ or } \frac{\dot{u}}{u} = 0, \qquad \text{(B11)} \end{aligned}$$

where (B10) follows from substituting with (18) and (B9).

From the definition of the production function in the final good sector, we get

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \tfrac{\dot{z}}{z} + (1-\alpha)\tfrac{\dot{A}}{A} + (1-\alpha)\tfrac{\dot{(1-u)}}{(1-u)} + \alpha g_K \qquad \text{(B12)} \\ \Leftrightarrow \left[(1-\alpha) + \frac{1}{\beta}\left(\frac{\epsilon+\delta}{1+\delta}\right)\right] g_K &= (1-\alpha)\tfrac{\dot{A}}{A} \text{ from (B11)} \\ \Leftrightarrow g_A &= \left[1 + \tfrac{1}{\beta(1-\alpha)}\left(\tfrac{\epsilon+\delta}{1+\delta}\right)\right] g_K. \qquad \text{(B13)} \end{aligned}$$

What remains is looking for a closed for solution for $g_c = g_Y = g_K$. This does indeed exist, and for this we proceed as follows.

Take (27) as one equation that defines the dynamics of λ_A. The other one is eq. (B8). Equating the two to eliminate $\frac{\dot{\lambda}_A}{\lambda_A}$, and substituting for $\frac{\dot{A}}{A}$ from (B13) yields

$$\begin{aligned} \rho - (\sigma-1)\eta L &= g_K - \epsilon g_K - \left[1 + \tfrac{1}{\beta(1-\alpha)}\left(\tfrac{\epsilon+\delta}{1+\delta}\right)\right] g_K \\ \Leftrightarrow g_K &= \frac{(\sigma-1)\eta L - \rho}{\epsilon + \frac{1}{\beta(1-\alpha)}\left(\frac{\epsilon+\delta}{1+\delta}\right)} = g_c = g_Y, \qquad \text{(B14)} \end{aligned}$$

which is the closed-form solution for the common growth rate of consumption per capita, capital stock and output of the final good along the balanced growth path. This could be used for deriving the specific parametric restrictions that ensure long-run sustainability of growth, which is discussed in Proposition 2.

Appendix C

In this appendix the behavior of key variables along the balanced growth path of the unregulated decentralized equilibrium is characterized.

To begin with, the solutions to r and u along the unregulated market equilibrium are derived. First, eq. (57) can be rearranged to yield,

$$\tilde{u} = \frac{\tilde{r} - \eta\alpha\sigma L - \frac{1}{(1-\alpha)}\tilde{g}_z}{-\eta L(\alpha\sigma + 1)}.$$

Next, (71) implies

$$\tilde{u} = \frac{\tilde{r} - \rho}{\epsilon(\sigma - 1)\eta L} - \frac{1}{(1-\alpha)} \frac{\tilde{g}_z}{(\sigma - 1)\eta L}.$$

Equating these two yields the following solutions

$$\tilde{r} = \frac{1}{(1-\alpha)} \tilde{g}_z \left[\frac{\epsilon(\sigma - 1) + \epsilon(\alpha\sigma + 1)}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \right] + \frac{\rho(\alpha\sigma + 1) + \eta\alpha\sigma L(\sigma - 1)}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)}, \tag{C1}$$

$$\tilde{u} = -\frac{1}{(1-\alpha)\eta L} \tilde{g}_z \left[\frac{\epsilon - 1}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \right] + \frac{1}{\eta L} \left[\frac{\eta\alpha\sigma L - \rho}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)}. \right] \tag{C2}$$

Next, by utilizing the consumption growth equation (65),

$$\begin{aligned} \tilde{g}_c = \tilde{g}_Y = \tilde{g}_K = \frac{\tilde{r} - \rho}{\epsilon} &= \frac{1}{(1-\alpha)} \tilde{g}_z + (\sigma - 1)\eta \tilde{u} L \\ &= \frac{1}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \left[\sigma \frac{(1+\alpha)}{(1-\alpha)} \tilde{g}_z + (\sigma - 1)(\eta\alpha\sigma L - \rho) \right]. \end{aligned} \tag{C3}$$

Furthermore, from (59), we have $\frac{\dot{E}}{E} = \frac{Y z^\beta}{E} - \theta$, which implies that, along the balanced growth path, $\frac{Y z^\beta}{E}$ will be constant, since $\frac{\dot{E}}{E}$ is constant and so is θ. Thus,

$$\begin{aligned} \tilde{g}_E &= \tilde{g}_Y + \beta \tilde{g}_z \\ &= \frac{1}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \left[\left(\sigma \frac{(1+\alpha)}{(1-\alpha)} + \beta \right) \tilde{g}_z + (\sigma - 1)(\eta\alpha\sigma L - \rho) \right]. \end{aligned} \tag{C4}$$

In the same vein, from (64), we have $\frac{\dot{\lambda}_e}{\lambda_e} = \delta \frac{\dot{E}}{E}$, since, in steady state, $\frac{\dot{\lambda}_e}{\lambda_e}$ will be constant, and so are ρ and θ. Thus,

$$\frac{\dot{\tilde{\lambda}}_e}{\tilde{\lambda}_e} = \frac{\delta}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \left[\left(\sigma \frac{(1+\alpha)}{(1-\alpha)} + \beta \right) \tilde{g}_z + (\sigma - 1)(\eta\alpha\sigma L - \rho) \right], \tag{C5}$$

and, from (63),

$$\frac{\dot{\tilde{\lambda}}_a}{\tilde{\lambda}_a} = \tilde{r} - \rho \equiv \frac{1}{\epsilon(\sigma - 1) + (\alpha\sigma + 1)} \left[\frac{(\epsilon(\sigma - 1) + \epsilon(\alpha\sigma + 1))}{(1-\alpha)} \tilde{g}_z + \epsilon(\sigma - 1)(\eta\alpha\sigma L - \rho) \right]. \tag{C6}$$

This completes the characterization of the unregulated market outcome.

Appendix D

In this appendix the behavior of key variables along the balanced growth path of the optimally regulated market equilibrium is characterized.

To begin with, the equilibrium values of r and u along the balanced growth path are derived. First, taking (88), we get

$$u' = \frac{r - \frac{1}{1-\alpha} g'_z - \eta \frac{\alpha}{(1-s_A)} \frac{\beta}{(1+\beta)} \sigma L}{-\eta L \left(\frac{\alpha}{(1-s_A)} \frac{\beta}{(1+\beta)} \sigma + 1 \right)}.$$

Next, (104) yields,

$$u' = \frac{r-\rho}{\epsilon(\sigma-1)\eta L} - \frac{1}{(1-\alpha)(\sigma-1)\eta L} g_z'.$$

By equating the two, we have the following solutions for r and u:

$$r' = \frac{1}{1-\alpha} g_z' \left[\frac{\epsilon(\sigma-1)+\epsilon(\alpha'\beta'\sigma+1)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right] + \left[\frac{\alpha'\beta'\eta\sigma L\epsilon(\sigma-1)+\rho(\alpha'\beta'\sigma+1)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right]; \tag{D1}$$

$$u' = -\frac{1}{(1-\alpha)\eta L} g_z' \left[\frac{\epsilon-1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right] + \frac{1}{\eta L} \left[\frac{\alpha'\beta'\eta\sigma L-\rho}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right], \tag{D2}$$

where $\alpha' = \frac{\alpha}{1-s_A}$ and $\beta' = \frac{\beta}{1+\beta}$.

Accordingly, the steady state rates of growth are worked out to be:

$$\begin{aligned} g_c' = g_Y' = g_K' = g_T' &= \frac{r'-\rho}{\epsilon} = \frac{1}{1-\alpha} g_z' + (\sigma-1)\eta u' L \\ &= \frac{1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \left[\sigma \frac{(\alpha'\beta'\sigma+1)}{(1-\alpha)} g_z' + (\sigma-1)(\alpha'\beta'\eta\sigma L-\rho) \right] \end{aligned} \tag{D3}$$

Further, from the constancy of $\frac{\dot{E}}{E}$ and θ when applied to (90), we get that

$$g_E' = g_Y' + \beta g_z', \tag{D4}$$

$$= \frac{1}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \left[\left(\sigma \frac{(\alpha'\beta'\sigma+1)}{(1-\alpha)} + \beta \right) g_z' + (\sigma-1)(\alpha'\beta'\eta\sigma L-\rho) \right]. \tag{D5}$$

Moreover, from (79),

$$\beta \frac{\dot{z}}{z} = -\frac{\dot{t_P}}{t_P} \Leftrightarrow g_z' = -\frac{1}{\beta} g_{t_P} \tag{D6}$$

Further, from the constancy of $\frac{\dot{\lambda}_e}{\lambda_e}$ along the balanced growth path as well as that of ρ, and θ in (96), we will have

$$\begin{aligned} \frac{\dot{\lambda}_e'}{\lambda_e'} &= \delta \frac{\dot{E}}{E} \equiv \delta g_E', \\ &= \frac{\delta}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \left[\left(\sigma \frac{(\alpha'\beta'\sigma+1)}{(1-\alpha)} + \beta \right) g_z' + (\sigma-1)(\alpha'\beta'\eta\sigma L-\rho) \right], \end{aligned}$$

and, from (95), it is derived that

$$\frac{\dot{\lambda}_a'}{\lambda_a'} = r' - \rho = \frac{1}{1-\alpha} g_z' \left[\frac{\epsilon(\sigma-1)+\epsilon(\alpha'\beta'\sigma+1)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right] + \left[\frac{\epsilon(\sigma-1)(\alpha'\beta'\eta\sigma L-\rho)}{\epsilon(\sigma-1)+(\alpha'\beta'\sigma+1)} \right]. \tag{D7}$$

This completes the characterization of steady state growth rates for the optimal regulation case.

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